Quantum-mechanics-free subsystem in a condensate-based optomechanical setup

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We propose a quantum optomechanical implementation of the quantum mechanics free subsystem (QMFS) proposed by Tsang and Caves [Phys. Rev. Lett. **105**, 123601 (2010); Phys. Rev. X **2**, 031016 (2012)] based on the mechanical properties of a Bose-Einstein condensate trapped in an optical lattice. In that scheme the effective mass of the atomic condensate is modulated by an optical lattice and can become negative, resulting in a negative-frequency optomechanical oscillator, negative environment temperature, and optomechanical properties opposite to those of a positive mass system. We also discuss possible protocols to apply this scheme to the back-action evading measurement of feeble fields.

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A fundamental issue faced by quantum metrology is the need to circumvent measurement back-action noise. Many of the early ideas were developed in the context of gravitational wave detection [1–4], with groundbreaking advances such as back-action evading – or quantum non-demolition (QND) – measurements, as well as to the realization of the potential benefits of using non-classical fields, such as squeezed states, to locate the unavoidable quantum fluctuations where they would not significantly perturb the measurement. Recently these ideas have witnessed a significant new development by groups around C. M. Caves [5] and E. Polzik [6], who realized that it is sometimes possible to isolate quantum-mechanics free subsystems (QMFS) of a quantum system. All observables in these subsystems are by construction QND observables, As such they may find a number of applications in the detection of feeble forces and fields, including optomechanical sensing, magnetometry and gravitational wave detection.

As discussed by Tsang and Caves [5], a simple setup to implement a QMFS comprises two harmonic oscillators of identical frequencies and opposite masses,

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{q}^2 - \frac{\hat{p}'^2}{2m} - \frac{1}{2}m\omega^2\hat{q}'^2. \tag{1}$$

Considering then the variables

$$\hat{Q} = \hat{q} + \hat{q}' \qquad \hat{P} = \frac{1}{2}(\hat{p} + \hat{p}')$$
$$\hat{\Phi} = \frac{1}{2}(\hat{q} - \hat{q}') \qquad \hat{\Pi} = \hat{p} - \hat{p}', \tag{2}$$

it is easily verified that

$$\dot{\hat{Q}}(t) = \frac{\hat{\Pi}(t)}{m} \qquad \dot{\hat{\Pi}} = -m\omega^2\hat{Q}(t). \tag{3}$$

Since $[\hat{Q}, \hat{\Pi}] = 0$ this means that the dynamical pair of observables $\{\hat{Q}, \hat{\Pi}\}$ formed by the collective position $\hat{Q}$ and relative momentum $\hat{\Pi}$ form a QMFS – and likewise for the pair $\{\hat{\Phi}, \hat{P}\}$. An implementation of this idea involving two spin ensembles oppositely polarized along an external magnetic field was recently demonstrated [6], and a system involving a cavity mode driven by a blue-detuned pump laser has also been advanced [7]. Here we propose a quantum optomechanical setup [8] where the effective negative mass system is produced by a Bose-Einstein condensate trapped in an optical lattice.

Cavity optomechanical systems based on the collective motion of Bose-Einstein condensates (BEC) [9] or non-degenerate ultracold atomic gases [10] have proven to be particularly well suited to demonstrate a number of quantum effects, including the observation of the quantum back-action of position measurements [10], the asymmetry in the power spectrum of displacement noise due to the non-cummutating nature of boson creation and annihilation [11], as well as the optomechanical cooling of a collective motional mode of an atomic ensemble down to the quantum regime [12]. These experiments pave the way to promising ultracold atoms-based quantum metrology schemes.

The concept of effective masses – which can in principle be negative as well as positive – is familiar from solid-state physics where it has proven useful in describing the motion of electrons in non-ideal lattice potentials [13]. Not surprisingly, that idea has recently been expanded to describe aspects of the dynamics of ultracold atoms in optical lattices [14, 15]. We show in this paper that when combined with BEC-based quantum optomechanics, it leads to the realization of QMFS that may find applications in the detection of feeble fields and forces.

Consider for concreteness a scalar atomic BEC confined by both an optical lattice potential $V_0\cos^2(k_L x)$ of periodicity $2\pi/k_L$ and an external trapping potential $U(x)$ that is taken to be slowly varying over the lattice period. Restricting the description to one dimension for simplicity, the condensate is described by the Hamiltonian

$$H = \int \hat{\Psi}^\dagger \left[-\frac{\hbar^2\nabla^2}{2m} + V_0\cos^2(k_L x) + U(x)\right]\hat{\Psi}dx, \tag{4}$$

where $\hat{\Psi}(x)$ is the bosonic field operator of the atomic sys-

tem and we have neglected inter-atomic collisions. Assuming that the lattice potential is sufficiently shallow that we are far from the Mott insulator transition [16] we proceed by expanding $\hat{\Psi}(x)$ in terms of a complete set of basis Bloch functions as

$$\hat{\Psi}(x) = \sum_n \int dq \phi_{n,q}(x) \hat{a}_{n,q} \tag{5}$$

where n and q label the band index and the quasi-momentum, respectively, and $\hat{a}_{n,q}$ are the associated boson annihilation operators. In the following we assume that the condensate is properly described by the product of an envelope that varies slowly over the period of the lattice, and is characterized by a central wave vector q_0, and Bloch functions that capture the rapid oscillations of the condensate caused by optical lattice. We then have approximately [17]

$$\phi_{n,q}(x) \approx e^{i(q-q_0)x} \phi_{n,q_0}(x) \tag{6}$$

where the mode functions $\phi_{n,q_0}(x)$ capture the density oscillations and

$$\hat{\Psi}(x) = \sqrt{2\pi} \sum_n \phi_{n,q_0}(x) \hat{\mathcal{A}}_{n,q_0}(x). \tag{7}$$

Here we have introduced the slowly varying bosonic operators (on the scale of the lattice period $2\pi/k_L$)

$$\hat{\mathcal{A}}_{n,q_0}(x) = (1/\sqrt{2\pi}) \int dq e^{i(q-q_0)x} \hat{a}_{n,q} \tag{8}$$

that describe the dynamics of the condensate envelope in the trapping potential $U(x)$, with

$$[\hat{\mathcal{A}}_{n,q_0}(x), \hat{\mathcal{A}}^\dagger_{n',q_0}(x')] = \delta_{n,n'} \delta(x - x'). \tag{9}$$

Applying the effective mass method [13] yields then the effective Hamiltonian describing the condensate in the envelope representation as

$$H_A = \sum_n \int \hat{\mathcal{A}}^\dagger_{n,q_0}(x) \Big[-\frac{\hbar^2 \nabla^2}{2m^*_{n,q_0}} + U(x) + \mathcal{E}_n(q_0) + \mathcal{E}'_n(q_0)\,(-i\nabla) \Big] \hat{\mathcal{A}}_{n,q_0}(x) dx, \tag{10}$$

where the effect of the lattice potential is now included in the single-particle effective mass

$$m^*_{n,q_0} = \hbar^2 / \mathcal{E}''_n(q_0). \tag{11}$$

Here $\mathcal{E}'_n(q_0)$ and $\mathcal{E}''_n(q_0)$ are the first and second-order derivatives of the nth-band Bloch energy with respect to the quasimomentum, evaluated at q_0. Figure 1 shows that the gradient term $\mathcal{E}'_n(q_0)$ vanishes at the center ($q_0 = 0$) an edges ($q_0 = \pm k_L$) of the first Brillouin zone.

The effective mass is negative at the zone edges for odd-n bands, and at the zone center for even-n bands. Note also that the potential $U(x)$ is shifted by the Bloch

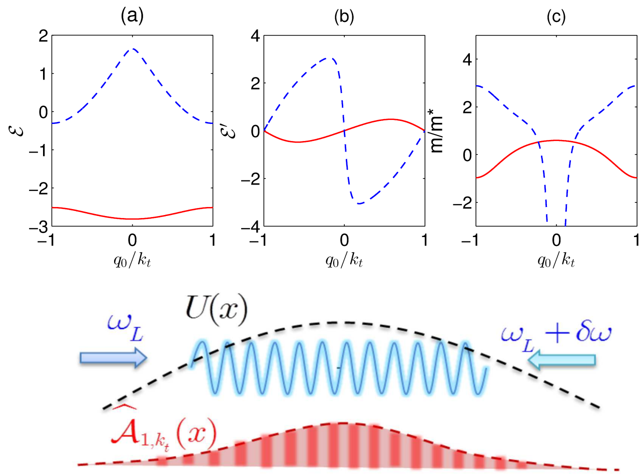

FIG. 1. Top: Bloch energy $\mathcal{E}(q_0)$ (a), its derivative (b), and the effective mass ratio (c) for a lattice depth $V_0 = 4.5E_r$ where $E_r = \hbar^2 k_L^2/2m$ is the atomic recoil energy. The red solid line and blue dashed lines are for the first and second band case, respectively. Bottom: sketch of the density profile of a negative effective mass condensate in a trap potential U(x): the density is modulated by the optical lattice and peaks at the maximum of U(x). "

energy $\mathcal{E}_n(q_0)$. For slightly deep lattice (still far from the Mott insulator transition) it is sufficient to consider the first band only. This is the situation that we consider in the remainder of this paper.

The validity of the negative effective mass description relies on the existence of a narrow momentum distribution of the condensate relative to the central wave vector $\pm k_L$. This can be achieved by applying an initial velocity to the condensate, or alternatively by considering a condensate initially at rest and adiabatically switching on a moving optical lattice realized by two counter-propagating fields of frequency difference $\delta\omega = 2\hbar k_L^2/m$. This permits to prepare the condensate with a specific quasi-momentum [15] such that the Hamiltonian (10) describes a quantum field of negative effective mass particles trapped in the potential $U(x) + \mathcal{E}_1(k_L)$.

The acceleration of particles with negative mass is opposite to the direction of the forces to which they are subjected, so that stability occurs at the maximum of a potential $U(x)$. We consider then the situation where a condensate of negative effective mass m^*_{1,k_L} is trapped in a potential $U(x)$ that we approximate as an inverted harmonic potential of frequency $\Omega = \sqrt{|U''(x_1)\mathcal{E}''_1(k_L)|}/\hbar$ with x_1 the position of its maximum. We expand the envelope field operator $\hat{\mathcal{A}}_{1,k_L}(x)$ on the basis of its eigenfunctions $\xi_\ell(x)$, with eigenenergies $\hbar\omega_\ell = -\hbar\Omega(\ell + \frac{1}{2}) < 0$, as

$$\hat{\mathcal{A}}_{1,k_L}(x) = \sum_\ell \xi_\ell(x) \hat{b}_\ell \tag{12}$$

where the bosonic operator $\hat{b}_\ell$ annihilates a condensed

atom from the mode ℓ.

In contrast with the situation for positive masses the "ground mode" $\xi_0(x)$, which has the largest population, has now the highest energy, But since the relative probability for a particle to occupy state ℓ is proportional to the Boltzmann factor $P_\ell \propto \exp[-\hbar\omega_\ell/k_B T]$, we conclude that the effective temperature T of a condensate with negative effective mass is also negative. We note that this situation is closely related to the scheme proposed in Ref. [18] and recently demonstrated by Braun *et al.* [19] to achieve negative temperatures. In that case a tight optical lattice was used to establish the density profile of the ultracold atoms initially trapped in a loose harmonic potential $U(x)$. That potential was then reversed from $U(x)$ to $-U(x)$, following which the low-energy states of $U(x)$ corresponded to the high-energy states of $-U(x)$. In our approach, in contrast, $U(x)$ is fixed and we move a shallow optical lattice to change the effective mass of the condensate from positive to negative.

From that point on, the realization of a negative mass optomechanical system follows closely the approach pioneered in Refs. [9, 10], with an optical cavity field of wave vector $k_c \ll k_L$ perturbing the center-of-mass motion of the condensate, but with important differences that we discuss in the following.

For short times the depletion of the ground mode $\xi_0(x)$ remains small and we can describe it classically, $\hat{b}_0 \approx \sqrt{N}$ with N the total atom number. The cavity-condensate coupling takes then the form of a multimode cavity optomechanical interaction

$$H = \hbar\omega_c \hat{c}^\dagger \hat{c} + \sum_\ell \left[G_\ell (\hat{b}_\ell + \hat{b}_\ell^\dagger) \hat{c}^\dagger \hat{c} + \hbar\omega_{M,\ell} \hat{b}_\ell^\dagger \hat{b}_\ell \right] \qquad (13)$$

where the effective optomechanical coupling coefficients are

$$G_\ell = \sqrt{N}\mathcal{D} \int \xi_0^*(x) \cos^2(k_c x - \theta) \xi_\ell(x) dx. \qquad (14)$$

Here $\mathcal{D}$ is the single-photon potential depth of the cavity field of frequency ω_c (including the frequency shift due to the condensate mean field) and annihilation operator $\hat{c}$, $\omega_{M,\ell} = \omega_\ell - \omega_0 < 0$ are the oscillation frequencies of the "effective condensate mirrors," [9] and θ is a phase that depends on the position of the maximum of $U(x)$ relative to the optical potential of the cavity field. We found numerically that coupling to the first excited mode $\ell = 1$ can be made dominant by an appropriate choice of that phase, the remaining modes acting as a mechanical reservoir for that mode [11]. (Other situations with another single dominant mode ℓ, or with two or more modes coupled with comparable strengths, can also be arranged.)

For the inverted harmonic trap considered here the bath temperature is negative, as already mentioned. One can thus expect a reversed asymmetric displacement spectrum $S_x(-\omega)$ for the negative frequency oscillator $\hat{b}_\ell$ [20] and hence a reversed optical output spectrum $n_c(-\omega)$ for the cavity field [11]. This means that the optomechanical properties of a negative effective mass oscillator are reversed form the usual case. For example optical cooling is realized by a blue-detuned rather than red-detuned driving field, and stationary bipartite entanglement is optimized in the red-detuned case [21], with the situation possibly even more interesting in the multimode case. When mechanical damping is weaker than the cavity decay, these negative frequency oscillators can serve as a negative temperature bath for a cavity field of positive frequency ω_c, so that it will exhibit gain as in usual laser theory – where a negative temperature is provided by the inversion of the active medium [22].

We are now in a position to discuss possible BEC optomechanical realizations of QMFS setups [5]. There are several ways to produce a pair of optomechanical oscillators with opposite effective masses. For example one could use a two-component condensate with only one component sensitive to the optical lattice, or excite the condensate to the edges of two adjacent Bloch bands. Two-component condensates allow for the sensing and measurement of fields that couple differently to the two components, such as e.g. magnetic fields coupled to a spinor condensate or gravitational waves propagating along the system.

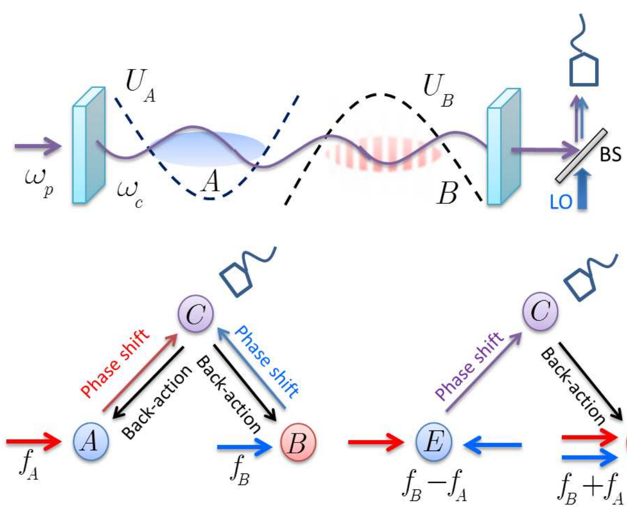

FIG. 2. Top: Possible arrangement for back-action-free field and force detection. Two condensates A and B are trapped along the axis of an optical cavity by the potentials U_A and U_B, respectively, with a moving optical lattice that drives condensate B and adjusts its effective mass to a negative value. Bottom: Relationship diagram for the setup in the "bare" (left) and "composite" (right) representations. The displacement of the composite subsystem D results in a change in the phase of the cavity field C that could be measured by homodyne detection, but the measurement back-action only affects the composite oscillator E.

Figure 2 illustrates the situation where two cigar-shaped condensates A and B are trapped along the x-

direction by the potentials U_A and U_B, respectively. A moving optical lattice interacts with the condensate B only and imparts it with a negative effective mass. The cavity resonance is sensitive to the motion of both condensates, which form an effective "two-mirror" cavity optomechanical system with one positive and one negative mass. A QMFS is realized provided that the trapping potentials U_A and U_B and the optical lattice depth V_0 are such that the optomechanical parameters of the two condensates are identical except for the sign of their masses, and hence optomechanical oscillator frequencies.

We assume for simplicity that a single mode is excited by the external perturbation to be measured, so that for each condensate a single term is dominant in Eq. (13). The effect of the weak coupling to other optical and envelope modes can be described by a standard fluctuation and dissipation approach. The Hamiltonian part of the system dynamics is then governed by

$$\begin{aligned} H &= \hbar\omega_c \hat{c}^\dagger \hat{c} + \hbar\omega_m(\hat{a}^\dagger\hat{a} - \hat{b}^\dagger\hat{b}) + G\hat{c}^\dagger\hat{c}(\hat{b} + \hat{b}^\dagger + \hat{a} + \hat{a}^\dagger) \\ &\quad + f_A(\hat{a} + \hat{a}^\dagger) + f_B(\hat{b} + \hat{b}^\dagger), \end{aligned} \tag{15}$$

where $\hat{a}$ and $\hat{b}$ are the mode annihilation operators for the condensates A and B of mechanical frequencies ω_M and $-\omega_M$, respectively, and G is the optomechanical coupling strength, see Eq. (13). The parameters f_A and f_B, which are time-dependent in general, describe the coupling of the external perturbation to the condensates.

The Heisenberg equations of motion for the cavity field and the composite displacement and momentum operators $\hat{Q} = (\hat{a} + \hat{a}^\dagger + \hat{b} + \hat{b}^\dagger)/2$ and $\hat{\Pi} = (\hat{a} - \hat{a}^\dagger - \hat{b} + \hat{b}^\dagger)/2i$ are

$$\begin{aligned} \dot{\hat{c}} &= i\Delta_c \hat{c} - \frac{2iG}{\hbar}\hat{Q}\hat{c}, \\ \dot{\hat{Q}} &= \omega_M \hat{\Pi}, \qquad \dot{\hat{\Pi}} = -\omega_M \hat{Q} + \frac{f_B - f_A}{\hbar}, \end{aligned} \tag{16}$$

where $\Delta_c = \omega_p - \omega_c$ is the cavity-pump detuning. These equations describe the motion of a particle driven by the difference between the strength of the external perturbations at condensates A and B. This causes a frequency shift of the cavity field that can be detected by interferometry. The key point here is that since these operators form a QMFS, $[\hat{Q}, \hat{\Pi}] = 0$, this measurement doesn't introduce any back-action to that QMFS, hence and the measurement is not subject to the standard quantum limit. Complementary conclusions are easily obtained for the second QMFS, characterized by the operators $\hat{\Phi} = (\hat{a} + \hat{a}^\dagger - \hat{b} - \hat{b}^\dagger)/2$ and $\hat{P} = (\hat{a} - \hat{a}^\dagger + \hat{b} - \hat{b}^\dagger)/2i$.

Further insight into the underlying physics of this back-action-free measurement scheme by considering the quantum state dynamics. We assume that the system is initially uncorrelated, with the cavity field is in a coherent state and the oscillators A and B both in the ground state,

$$|\psi(0)\rangle = |\alpha\rangle_C \otimes |0\rangle_A \otimes |0\rangle_B. \tag{17}$$

As a result of the optomechanical interaction (15) the oscillators A and B become entangled with the cavity field C. However, when expressing the state of the system in terms of the composite oscillators D and E described by the operators $\{\hat{Q}, \hat{P}\}$ and $\{\hat{\Phi}, \hat{\Pi}\}$, respectively, we find that the system does not suffer three-body entanglement between the subsystems C, D and E, but only two-body entanglement. Specifically we find, except for an unimportant constant phase factor,

$$\begin{aligned} |\psi(t)\rangle &= e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}} \exp\left[\frac{i2nGq_s}{\hbar\omega_M}(\omega_M t - \sin\omega_M t)\right] \\ &\quad \times |n\rangle_C |\phi_n(t)\rangle_D \otimes |\varphi(t)\rangle_E, \end{aligned} \tag{18}$$

where

$$\begin{aligned} \phi_n(t) &= \frac{-1}{\sqrt{2}\hbar\omega_M}(f_A + f_B + 2Gn)\left(1 - e^{-i\omega_M t}\right), \\ \varphi(t) &= \frac{-1}{\sqrt{2}\hbar\omega_M}(f_A - f_B)\left(1 - e^{-i\omega_M t}\right), \end{aligned} \tag{19}$$

and $q_s = (f_B - f_A)/\hbar\omega_M$. Equation (18) shows that in contrast to the composite oscillator D, which becomes entangled with the cavity mode C, the composite oscillator E remains uncorrelated to the rest of the system. Rather, it evolves into a time-dependent coherent sate $|\varphi(t)\rangle_E$ that is independent of both the state of the optical field and the composite oscillator D. Its complex amplitude is fully determined by the steady displacement (16) and the frequency ω_M.

The states $\phi_n(t)\rangle_D$ are n-dependent coherent states of the composite oscillator D. They are similar to those encountered in single-mirror optomechanical situations [23], except for the important difference that the phase factor dependence depends on the steady-state displacement q_s of the oscillator E. That dependence makes it is easy to read out q_s without measurement back-action. For example for $\omega_M t = 2m\pi, n$ integer, the state of the system reduces to $|\alpha \exp[-4im\pi G q_s]\rangle_C \otimes |0\rangle_D \otimes |0\rangle_E$, that is, the composite oscillator D and E return to vacuum state – as do the oscillator A and B – while the cavity field becomes a coherent state whose phase could be easily measured by homodyne detection.

Summarizing, we have proposed and analyzed a BEC-based optomechanical system that permits to isolate a QMFS with potential applications in back-action free measurements of feeble forces and fields. Our discussion centered on the use of quantum-degenerate atomic systems whose spatial extent is large to the period of the optical lattice. One might ask if it is also possible to achieve a QMFS in ultracold, but non-condensed atoms. In that case every atom is localized within a single lattice well, but the phonon mode associated with the center of mass of the sample can still be delocalized and can enter the quantum regime via cavity optomechanical cooling [12]. However thermal damping is now of the order of $\gamma \sim \omega_M$

and is expected to significantly affect the measurement precision of the scheme. Future work will discuss in detail the effects of dissipation in the proposed system and consider specific applications, including magnetometry and gravitational wave detection.

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